Abstracts of Talks
Workshop on Algorithms, Combinatorics, and Geometry

University of North Texas, College of Engineering
November 29 to December 1, 2007

Edited by Cameron Lowell Palmer

Thursday, November 29, 2007, Morning Session

Turán-Type Results on Intersection Graphs of Geometric Objects

János Pach - City College and Courant Institute (New York)

Which graphs are intersection graphs of convex sets, continuous arcs, rectangles, segments or other geometric objects in the plane? Most of these questions are algorithmically hard. In many cases it is not even clear whether they are decidable. However, most classes of intersection graphs have very special extremal graph-theoretic properties. We concentrate on Turán-type results.

According to the Kővári–Sós–Turán theorem, any graph with n vertices that contains no $K_{k,k}$ (no complete bipartite subgraph with k vertices in each of its classes), has $O(n^{2-1/k})$ edges. However, if G is the intersection graph of n compact connected sets $C_1, \ldots, C_n$ in the plane, then the under the same assumption the number of edges of G can be bounded from above by n times a polylogarithmic factor. Here the exponent of the logarithmic factor depends on k. Moreover, if each C_i is convex, then the number of edges is $O(n)$. For the proofs we need some results from geometric graph theiry and various extensions of the Lipton-Tarjan separator theorem for planar graphs.

Joint work with Micha Sharir and Jacob Fox.

Coloring Axis-Parallel Rectangles

Gábor Tardos - Simon Fraser and Rényi Institute

For every k and r, we construct a finite family of axis-parallel rectangles in the plane such that no matter how we color them with k colors, there exists a point covered by precisely r members of the family, all of which have the same color. In fact, to avoid such a point when coloring n rectangles you need to use $k = \Omega(\log n/(r \log r))$ colors.

This shows the tightness of the result of Shakhar Smorodinsky stating that $O(\log n)$ colors are enough when coloring axis-parallel rectangles to make sure that all pints covered by more than one rectangles will not be covered by only rectangles of a single color.

While we could not achieve such tight results in higher dimensions we can construct an arrangement of n axis-parallel boxes in d-dimension that require $\Omega(\log^{\lfloor d/4 \rfloor} n)$ colors if points covered by several boxes, all of the same color are to be avoided.

The results represent joint work with János Pach and Shakhar Smorodinsky.

Minimum Weight Convex Steiner Partitions

Csaba D. Tóth - University of Calgary

New tight bounds are presented on the minimum length of planar straight line graphs connecting n given points in the plane and having convex faces (specifically, every bounded face is convex, and the unbounded face is the complement of a convex polygon). We show that the convex Steiner partition of n points in the plane is at most $O(\log n / \log \log n)$ times longer than their Euclidean minimum spanning tree (EMST), and this bound is best possible. Without allowing Steiner points, the corresponding bound is known to be $\Theta(\log n)$, attained for n points lying along a pseudo-triangle. We also show that the convex Steiner partition of n points along a pseudo-triangle is at most $O(\log \log n)$ times longer than the EMST, and this bound is also best possible. Our methods are constructive and lead to polynomial-time algorithms for computing convex Steiner partitions within these bounds in both cases.

Joint work with Adrian Dumitrescu.

Intersecting Convex Sets By Rays

Andreas Holmsen - University of Bergen and NYU

What is the smallest number $\tau = \tau(n)$ such that for any collection of n pairwise disjoint convex sets in $\mathbb{R}^d$, there is a point such that any ray (half-line) emanating from it meets at most τ sets of the collection? This question is closely related to the notion of regression depth introduced by Rousseeuw and Hubert (1996). We will show the following.

Theorem 1. *Given any collection C of n pairwise disjoint compact convex sets in $\mathbb{R}^d$, there exists a point $p \in \mathbb{R}^d$ such that any ray emanating from p meets at most $\frac{dn+1}{d+1}$ members of C.*

Furthermore, we show that for $d = 2$, the above bound is tight apart from an additive constant.

Theorem 2. *Let n be a multiple of 3. There exist collections of n pairwise disjoint (i) disks or (ii) equal length segments in the Euclidean plane such that from any point there is a ray that meets at least $\frac{2n}{3} - 2$ of them.*

Joint work with Rados Fulek and János Pach.

Thursday, November 29, 2007, Afternoon Session

Parallel Repetition Of The Odd Cycle Game

Mario Szegedy - Rutgers University

The Odd Cycle Game is a two-prover game where the provers, Alice and Bob, both color their own copy of the odd cycle C_n with two colors. They optimize their coloring, for the following verification procedure: The verifier picks a random $0 \leq i \leq n-1$ and a random type from $0, 1$. In the type 0 test the verifier accepts if Alice and Bob color node i in the same way, otherwise rejects. In the type 1 test the verifier accepts if Alice's color of node i is different Bob's color of node $i+1(\mod n)$, otherwise rejects. It is easy to see that if they play optimally, Alice and Bob can win with probability $1 - 1/2n$. This is called the value of the game. The value of the powers (i.e. its parallel repeated versions) of the game has been the focus of recent investigations. We

1. Determine the exact value of the square of the game;
2. Show that if Alice and Bob are allowed to communicate a few bits they have a strategy with greatly increased winning probability;
3. Develop a new metric conjectured to give the precise value of the game up-to second order precision in $1/n$ for constant d and also with high precision for larger ds.
4. Show an $1 - \Omega(d/n \log n)$ upper bound for $d \leq n \log n$ if one player uses a "symmetric" strategy.
5. Give some insight into the $d = 3$ case.

Joint work with Kooshiar Azimian.

Communicating In An Unknown Game

Thomas Böhme - Technische Universität Ilmenau - thomas.boehme@tu-ilmenau.de

Consider two agents 1,2 playing a game. In each round agent i can choose an action a_i from a finite set A_i. Once both agents have chosen their actions a_1, a_2 agent i receives a signal $f_i(a_1, a_2)$ ($i \in \{1, 2\}$). It is assumed that

- prior to the beginning of the game the agents do not know the game, i.e. neither the sets A_1, A_2 nor the functions f_1, f_2, and

- once the game has started the agents cannot communicate with each other or observe each other.

We study the question whether the agents can utilize such a game to transmit messages. The answer to this question depends on whether the agents can communicate prior to the beginning of the game, and on the nature of the functions f_1, f_2. (If, for example, both functions are constant, then it is clearly impossible to transmit a message.)

It is proved that

(a) if the agents are allowed to agree on an algorithm prior to the beginning of the game, they can establish a communication under some mild conditions on the functions f_1, f_2, and

(b) if they cannot communicate prior to the beginning of the game, they cannot establish a communication.

We will also discuss generalizations of these results to the case where more than two agents are involved.

The results presented in this talk are joint with Jens Schreyer and Patrick Bauermann (TU Ilmenau).

Discharging And Reducibility: An Introduction

Daniel Cranston - DIMACS, Rutgers University

Many coloring results and structural results on graphs are proved using a pair of techniques called reducibility and discharging. To prove a hypothetical Theorem A for planar graphs, our proof has two phases: a discharging phase and a reducibility phase. In the discharging phase, we prove that every graph in a particular family (for example: planar graphs) must contain at least one subgraph from a specified list. The name *discharging* comes from the fact that we assign a charge (an integer) to each vertex so the sum of the charges is negative; we move the charge around (discharge it) so that only vertices appearing in one of the specified subgraphs have negative charge. In the reducibility phase, we prove that any counterexample to Theorem A (or oftentimes, a counterexample to Theorem A with the fewest vertices) cannot contain any subgraph in the specified list. This implies that there is no counterexample to Theorem A, and hence, the theorem is true. The most well-known proof that uses discharging and reducibility is the 4-Color Theorem.

In this talk, we illustrate the techniques of discharging and reducibility by proving the following theorem: The vertices of a planar graph of girth at least 14 can be partitioned into two subsets I and F such that G[F] is a forest and I is a 2-independent set, that is: any two vertices in I are distance at least 3 apart. This structural theorem has applications to *star coloring*; a star coloring is a proper coloring such that the union of any two color classes induces a forest of disjoint stars. This is joint work with Craig Timmons and André Kündgen, both of Cal State, San Marcos.

On Stars And Steiner Stars

Adrian Dumitrescu - University of Wisconsin-Milwaukee

For a set of n points in the plane, a *star* connects one of the points (the center) to the other $n-1$ points by straight line edges, while a *Steiner star* connects an arbitrary point in the plane to all n input points. The center of the *minimum* Steiner star, the *Weber center*, minimizes the sum of distances from the n points. Fekete and Meijer showed that the minimum star is at most $\sqrt{2}$ times longer than the minimum Steiner star for any finite point configuration in the plane or 3-space. The maximum ratio between the two is conjectured to be $4/\pi$ in the plane and $4/3$ in three dimensions. Here we improve the upper bound to 1.3999 in the plane, and to $\sqrt{2}-10^{-4}$ in 3-space. Our results also imply improved bounds on the maximum ratios between the minimum star and the maximum matching in two and three dimensions. Our method relies on constructing a suitable discretization for a continuous problem and then using *linear programming* to optimize over a relatively large set of constraints.

This is joint work with Csaba Tóth.

Friday, November 30, 2007, Morning Session

Authentication Of Outsourced Combinatorial And Geometric Structures

Roberto Tamassia - Brown University

The outsourcing of data and computations is a growing trend in information technology. We present methods for efficiently verifying the integrity of data stored by an untrusted responder on behalf of the data owner. In reply to a query from a client, the responder computes and returns the answer plus a succinct proof of it. The client then verifies the proof by relying solely on the trust in the data owner. We focus on authentication techniques for queries on combinatorial and geometric structures, such as trees, graphs, planar subdivisions, and point sets. An important component of the methods presented is a directed acyclic graph associated with a hierarchical cryptographic hash computation. This work has applications to the authentication of outsourced file systems, databases, and geographic information systems.

Simultaneous Graph Embeddings

Stephen Kobourov - University of Arizona

Traditional problems in graph visualization involve the layout of a single graph, while problems in simultaneous graph visualization involve the layout of multiple related graphs. A series of related graphs may arise from one relation between a set of objects as it evolves through time or from several relationships defined on the same set of objects. In simultaneous graph embedding, nodes are placed in the exact same locations in all the graphs and a series of graphs are simultaneously embedable if it is possible to find locations that yield straight-line crossing-free drawings for each of the individual graphs. We present polynomial time algorithms for simultaneous embedding of various classes of planar graphs and prove that some classes of graphs cannot be simultaneously embedded. Further, we present a near-linear time algorithm for visualizing graphs that evolve through time and demonstrate its application to problems in software engineering and databases.

Minimizing Crossings In Several Graphs Simultaneously

Michael Schulz - University of Cologne - schulz@informatik.uni-koeln.de

We study the crossing minimization problem in the context of simultaneous graph drawing. There is a natural way to extend the concept of crossing minimization from a single graph to multiple graphs that arise in simultaneous graph drawing. We define a new crossing number for this setting and discuss differences with the ordinary crossing number problem. We show how traditional crossing minimization algorithms can be adapted to applications where more than one graph is given at a time.

Distance-Avoiding Sets For Low-Power Authentication

Michael J. Collins - Sandia National Laboratories - mjcolli@sandia.gov

We consider several variations on the following problem: given q, find $S \subset \mathbb{N}$ such that for each $d > 0$, there are at least q integers i with $i \in S$, $i + d \notin S$, minimizing $max(S)$.

These problems arise in the context of secure communication for very low-power devices such as "smart dust". We wish to cryptographically authenticate a stream of many short messages $m_1, m_2, m_3, \cdots$ on a channel with limited bandwidth and a high rate of dropped messages. Because bandwidth is so constrained, we must use almost all transmitted bits for delivering payload: say we can append no more than r bits of authentication to each message, where r is too small to give adequate security.

We cannot simply group messages together and append a full length authentication tag to each group; most groups would include dropped messages, making it impossible to verify their tags. We also seek to avoid the cost of applying an error-correction scheme. Our approach is to append an r-bit tag a_i to each message m_i ; each a_i authenticates some appropriately-chosen subset S_i of the previous messages. If each message appears in q sets, then each message is used in the computation of q different tags, and we eventually accumulate qr bits of authentication for each message (unique sequence numbers prevent an adversary from attacking each tag separately).

If we require that for each pair (m_i, m_j), message m_i must appear in at least q sets that do not contain m_j, then loss of one message m_j will not prevent full authentication of any other m_i. With more general conditions, we can tolerate the loss of multiple messages, or we can limit the number of messages which are invalidated by the loss of another. We seek to minimize the number of old messages which the sender must remember, which leads to the aforementioned optimization problems.

Folding And Spiralling: The Word View

Marcus Schaefer - DePaul University - mschaefer@cs.depaul.edu

We show that for every n there are two simple curves on the torus intersecting at least n times without the two curves folding or spiralling with respect to each other.

Friday, November 30, 2007, Afternoon Session

Algorithms On Graphs Embedded On Surfaces

Bojan Mohar - Simon Fraser University

Some hard computational problems on graphs admit efficient solution when restricted to classes with additional information. Among the most important such classes is the class of all planar graphs. There are natural generalizations, including graphs of bounded genus or graphs in some proper minor-closed family. When dealing with these families of graphs, several basic computational problems stand out by their importance. For example, one wants to find a shortest non-contractible cycle in a graph embedded in a closed surface. Efficient solutions to this one and a myriad of similar problems will be discussed, and various algorithmic applications will be presented.

Joint work with Eric Sedgwick and Daniel Štefankovič

Ramsey-Type Results For Intersection Graphs Of Geometric Objects

Jacob Fox - Princeton University

Classical results in Ramsey theory and extremal combinatorics demonstrate the existence of patterns in graphs under rather mild constraints. However, these results do not give satisfactory answers to many of the fundamental questions concerning intersection graphs of geometric objects. With the development of new separator theorems, connections with partially ordered sets, extremal results for planar intersection graphs, and probabilistic techniques, it is now possible to prove several conjectures and striking results demonstrating the existence of surprisingly large homogeneous intersection patterns in arrangements of geometric objects. For example, there is $c > 0$ such that in every family of n convex sets in the plane, there are two subfamilies of size at least cn such that either each element of the first intersects every element in the second, or no element in the first intersects any element of the second.

Joint work with János Pach, Benny Sudakov, and Csaba D. Tóth.

Linear Bound On Extremal Functions Of Some Forbidden Patterns In 0-1 Matrices

Rados Fulek - Simon Fraser University - rfulek@sfu.ca

$$L_1 = \begin{pmatrix} & \bullet & \bullet & \\ \bullet & & & \bullet \\ & \bullet & & \end{pmatrix} \qquad L_2 = \begin{pmatrix} & \bullet & \bullet & \bullet & \\ \bullet & & & & \bullet \\ & & \bullet & & \end{pmatrix}$$

Figure 1: **Patterns L_1 and L_2**

By saying that a 0-1 matrix A avoids a pattern P given as a 0-1 matrix we mean that no submatrix of A either equals P or can be transformed into P by replacing some 1 entries with 0 entries. We present a new method for estimating the maximal number of the 1 entries in a matrix that avoids certain patterns. Applying this method we give a linear bound on the maximal number $\mathrm{ex}(n, L_1)$ of the 1 entries in an n by n matrix avoiding pattern L_1 (Figure 1) and thereby we answer the question that was asked by Tardos (2005). Furthermore, we use our approach on one other pattern L_2 (Figure 1).

Drawing Cubic Graphs With Four Slopes

Padmini Mukkamala - Rutgers University

A straight-line drawing of a graph G is one where the vertices of G are represented by distinct points in the plane and every edge is represented by a straight-line segment connecting the corresponding pair of vertices and not passing through any other vertex of G. If we consider the problem of minimizing the number of distinct slopes in a straight line drawing of a graph, it was shown that graphs with maximum degree 5 cannot be drawn with a finite number of slopes and that five slopes suffice for connected cubic graphs. I will show that four slopes are enough in the later case.

Joint work with Mario Szegedy.

Saturday, December 1, 2007, Morning Session

Arithmetic Illiteracy: Algorithms We Were Not Told We Knew

David Matula - Southern Methodist University

What are the most efficient algorithms and numeric data structures for fundamental arithmetic operations in current day hardware arithmetic logic units (ALUs)? Most cultures teach grade school subtract with borrow, long-multiplication (with digit product tables), long-division and square-root. Students often equate these procedures with the operations, and the notion of comparative algorithms for addition, multiplication, division, and rounding of results is deferred to graduate school. So what is the current hardware algorithm design state?

On Representation: Users need decimal display, data storage requires binary, and the ALU requires redundant binary. The latter is uncomfortable even for ALU designers (e.g. the Intel divide bug problem)

On Multiplication: Considered a "solved" problem in general using Booth recoding, 3-2 adders, and Wallace tree reduction. These simple breakthrough ideas are unfortunately not taught alongside the comparable breakthrough ideas of doubly linked lists and heaps for data management, so arithmetic remains an arcane art, understood by few and misdesigned by many.

On Division: Effectively an unsolved problem of hardware algorithm design, despite the advances of Convergence Division and Bipartite Tables.

This talk presents the core algorithms from the author's dual perspectives of (i) algorithmic complexity, and (ii) ALU design as a codesigner of commercially sucessful 1990's era IEEE standard floating point (x87) coprocessors and more recently as a codesigner of the ALU for the one-watt geode processor chosen for the One Laptop Per Child (OLPC) project.

Improved Algorithms For One Sided Two-Layer Planarization Problem And Sensor Networking

Fenghui Zhang - Texas A&M University - fhzhang@cs.tamu.edu

The one sided two-layer planarization problem is defined as follows.

Given a bipartite graph $B(L, R)$ of n vertices with vertices in L fixed on a straight line l_1, remove the minimum number of edges in B such that vertices in R can be arranged on a parallel line l_2 of l_1 and no two edges of B cross each other.

This problem is closely related to the very essential one sided two-layer cross minimization problem in Circuit Design field. It has been proved to be NP-hard even when the degree of the vertices in R is bounded by 2. Previous best approximation algorithm for this problem gives ratio 3. When the number k of edges to be removed is taken as a parameter, the best known exact algorithm for this problem runs in time $O(3^k P(n))$ where $P(n)$ is a polynomial function of n.

We present improved approximation algorithm and exact algorithm for this problem. Our approximation algorithm has ratio $(2+\epsilon)$ where ϵ is any positive number. Our exact algorithm runs in time $O((2+\epsilon)^k P(n))$.

Network planarization is essential and hard for unlocalized sensor networks. Based on our algorithms for the one sided two-layer planarization problem we develop robust planarization algorithms for such sensor networks. We examine the practical performance of our algorithms in extensive simulations.

Joint work with Anxiao(Andrew) Jiang and Jianer Chen

Towards An Optimal Algorithm For Recognizing Laman Graphs

Ovidiu Dasescu - University of Texas At Dallas - daescu@utdallas.edu

Laman graphs are fundamental to rigidity theory and are closely related to *pointed pseudo-triangulations* of planar point sets. A graph G with n vertices and m edges is a *Laman graph*, or equivalently a generically minimally rigid graph, if $m = 2n - 3$ and every induced subset of k vertices spans at most $2k - 3$ edges.

Given a graph G with n vertices, the **Verification problem** asks to decide if G is Laman. The best known algorithm for this problem takes $O(n^{3/2})$ time. In this talk we will present a new algorithm. The algorithm exploits a known construction called red-black hierarchy, that is a certificate for Laman graphs and can be verified in linear time.

Joint work with Anastasia Kurdia

Saturday, December 1, 2007, Afternoon Session

RNA-RNA Interaction Algorithms

Saad Mneimneh - Hunter College of CUNY

The interaction of two RNA molecules is a common mechanism for many biological processes. Therefore, algorithms that predict the structure of an RNA complex are of great interest. The RNA-RNA interaction problem can be modeled as a weighted graph where, given a particular drawing of the graph, we seek a set of nonintersecting edges with the largest weight.

Most of the proposed algorithms to solve this problem are based on dynamic programming. RNA-RNA interaction is generally NP-complete; therefore, these algorithms (and other polynomial time algorithms for that matter) are not expected to produce optimal structures. Our goal is to characterize this sub optimality.

We demonstrate the existence of constant factor approximation algorithms that are based on dynamic programming. In particular, we describe 1/2 and 2/3 factor approximation algorithms.

We also define an entangler and prove that 2/3 is a theoretical upper bound on the approximation factor of algorithms that produce entangler free solutions, e.g. the mentioned dynamic programming algorithms.

Knowledge States With An Application To Cross Polytope Spaces

Wolfgang Bein - University of Nevada, Las Vegas - bein@cs.unlv.edu

We introduce the novel concept of knowledge states; many well-known algorithms can be viewed as knowledge state algorithms. The knowledge state approach can be used to to construct competitive randomized online algorithms and study the tradeoff between competitiveness and memory. As an example of our method we consider here the 2-server problem over Cross Polytope Spaces $M_{2,4}$. We describe an algorithm with competitive ratio of $\frac{19}{12}$, a ratio best possible.

Joint work with K. Iwama (Kyoto University) and L. Larmore (UNLV).

Schreier Spectrum of the Hanoi Towers Group on Three Pegs

Zoran Šunić - Texas A&M University - sunic@math.tamu.edu

Finite dimensional representations of the Hanoi Towers group are used to calculate the spectra of the finite graphs associated to the Hanoi Towers Game on three pegs. These graphs are Schreier graphs of the action of the Hanoi Towers group on the levels of the rooted ternary tree (the graph of the action on level 3 is given in Figure 2). The spectrum of the limiting graph (Schreir graph of the action on the boundary of the tree) is also provided. The spectrum has the form $I \cup J$, where J is a Cantor set of points (it is the Julia set of the polynomial $f(x) = x^2 - x - 3$) and I is a countable set of isolated points that accumulates to J. This is a joint work with R. Grigorchuk.

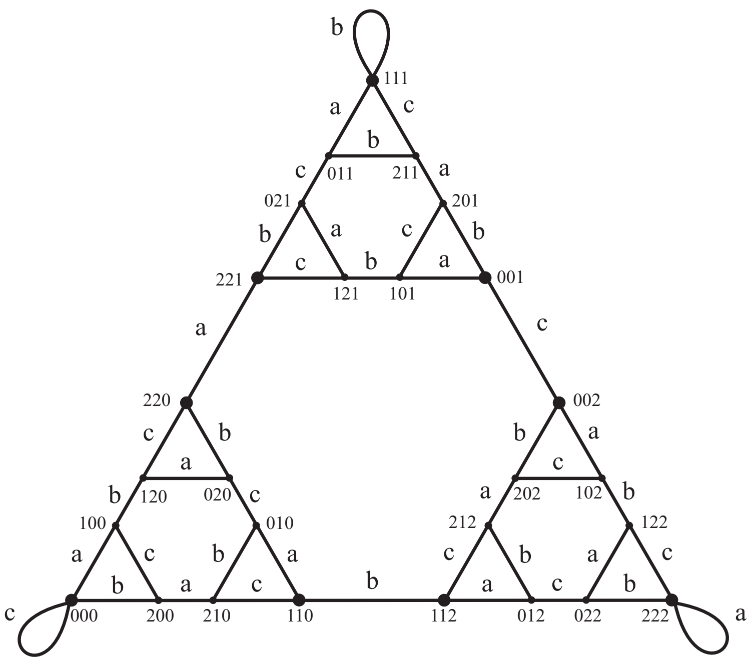

Figure 2: Γ_3, the Shcreier graph of H at level 3